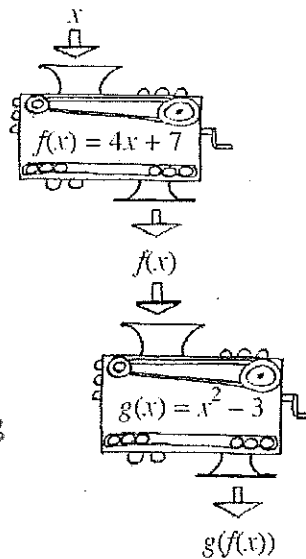


Composition of Functions Notes

Composing functions means that the output of one function becomes the input for another function. This can be thought of as stacking functions, as shown in the diagram at right. The top function's output becomes the bottom function's input. The composition in this diagram is expressed as $g(f(x))$ and can also be written as $(g \circ f)(x)$.



Suppose 1 is dropped into the machine f , and that output is dropped in g . We can represent this by $g(f(1))$. What is the value of $g(f(1))$? (That is, what is the final output?)

$$g(f(x)) = (4x+7)^2 - 3$$

$$g(f(1)) = (4(1)+7)^2 - 3$$

$$g(f(1)) = (11)^2 - 3$$

$$g(f(1)) = 121 - 3$$

$$g(f(1)) = 118$$

Evaluate $g(f(-0.5))$.

$$g(f(-0.5)) = (4(-0.5)+7)^2 - 3$$

$$g(f(-0.5)) = (-2+7)^2 - 3$$

$$g(f(-0.5)) = (5)^2 - 3$$

$$g(f(-0.5)) = 25 - 3$$

$$g(f(-0.5)) = 22$$

Evaluate $g(f(a))$, where a is a constant.

$$g(f(a)) = (4a+7)^2 - 3$$

$$g(f(a)) = 16a^2 + 56a + 49 - 3$$

$$g(f(a)) = 16a^2 + 56a + 46$$

Write an equation for $f(g(x))$.

$$f(g(x)) = 4(x^2 - 3) + 7$$

$$f(g(x)) = 4x^2 - 12 + 7$$

$$f(g(x)) = 4x^2 - 5$$

Write an equation for $g(f(x+1))$.

$$g(f(x+1)) = (4(x+1)+7)^2 - 3$$

$$g(f(x+1)) = (4x+4+7)^2 - 3$$

$$g(f(x+1)) = (4x+11)^2 - 3$$

$$g(f(x+1)) = 16x^2 + 88x + 121 - 3$$

$$g(f(x+1)) = 16x^2 + 88x + 118$$

Does the order that the function machines are stacked matter? That is, in general, does $f(g(x)) = g(f(x))$? Explain and provide examples or counterexamples to justify your decision.

Order does matter unless the functions are inverses. Example: $g(f(1)) = 118$

$$f(g(1)) = 4(1^2 - 3) + 7$$

$$f(g(1)) = 4(-2) + 7$$

$$f(g(1)) = -8 + 7$$

$$f(g(1)) = -1$$

$$118 \neq -1$$