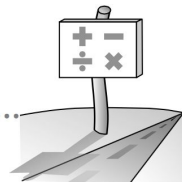


3.1.1 Think rationally.

Operations with Rational Expressions



#1 Simplify the expression:

$$\frac{x-3}{x-4} - \frac{x-3}{x-6}$$

#2 Which of the following pairs of expressions in parts (a) and (b) are equivalent? Justify your answers.

a.

$$\frac{x^2+4x+3}{4x+3} \text{ and } x^2, \text{ for } x \neq -\frac{3}{4}$$

b.

$$\frac{(x+2)(4x+3)}{4x+3} \text{ and } x+2, \text{ for } x \neq -\frac{3}{4}$$

c. Why is $x = -\frac{3}{4}$ excluded from the expressions in parts (a) and (b)?

#3 Simplify each of the following rational expressions. State any value(s) of the variable that need to be excluded.

i.

$$\frac{(2x-7)^6}{(x+1)(2x-7)^2}$$

ii.

$$\frac{4x^2-11x-3}{x^2-x-6}$$

iii.

$$\frac{3-5x}{(3x+1)(5x-3)}$$

#4 Multiply or divide each of the following expressions. Simplify the result, if possible. State the values of the variable that need to be excluded.

a.

$$\frac{12(x+9)^4}{8x} \cdot \frac{x^2+5x}{(x+9)^{12}}$$

b.

$$\frac{5x^2-14x-3}{x^2-9} \div \frac{5x^2+6x+1}{2x^2+x-1}$$

#5 Is $\frac{3x}{2x+5} + \frac{12}{2x+5}$ equivalent to $\frac{3x+12}{4x+10}$? Justify your answer.

#6 Add or subtract each of the following rational expressions. Simplify the result, if possible. State the value(s) of the variable that need to be excluded.

a.

$$\frac{5x}{x+3} - 4$$

b.

$$\frac{4x^2+5x-5}{16x^2-1} - \frac{x-2}{4x-1}$$

c.

$$\frac{7}{4x^3} + \frac{5}{6x} + \frac{2}{3x^4}$$

#10 Now it is time to create an addition problem by working backwards. If the sum of two fractions, each of which has a linear expression in the denominator, is $\frac{15x+19}{(2x+3)(x+1)}$, what are the two fractions?

a. What is the denominator of each fraction?

b. Write an addition problem with two fractions using A and B as the numerators and the denominators from part (a).

c. Now write an equation using $\frac{15x+19}{(2x+3)(x+1)}$ and your expression from part (b).

d. How can you rewrite the equation so that the denominators are eliminated? Discuss this with your team and then rewrite the equation without fractions.

e. Now write and solve a system of equations containing A , B , 15, and 19. Hint: The sum of the coefficients of x is 15, and the constant term is 19.

f. What are two fractions whose sum is $\frac{15x+19}{(2x+3)(x+1)}$?

G. Verify that your expression from part (e) is equivalent to $\frac{15x+19}{(2x+3)(x+1)}$.

#11 Use the ideas from problem 3-10 to determine the partial fraction decomposition of

$$\frac{29x-5}{3x^2-20x-7} .$$