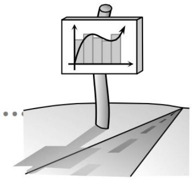
CW#____ Name:____________________________

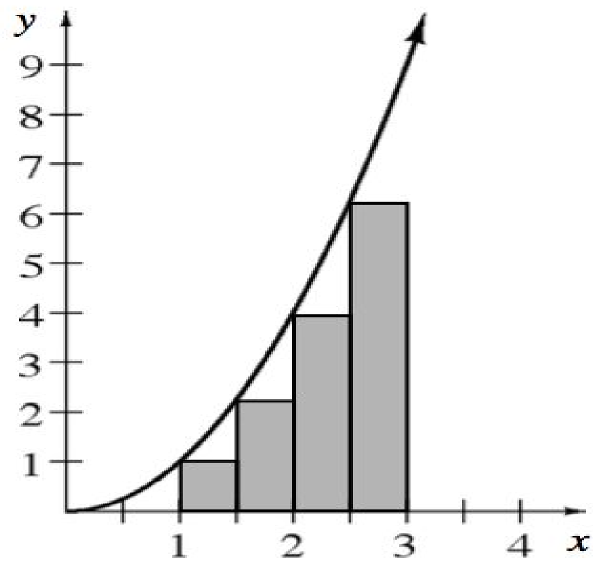
3.2.3 How can I approximate the area?

Area Under a Curve: Part Two



#114 The graph of $f(x) = x^2$ for $x \geq 0$ is shown below. Notice that the interval, $1 \leq x \leq 3$, is divided into four equal sub-intervals.

a, Use the equation $f(x) = x^2$ to obtain the coordinates of the points where the rectangles touch the curve. Label the points on your graph.



b. In terms of the rectangles drawn, what does the *y*-value of each coordinate represent?

c. What is the width of each rectangle?

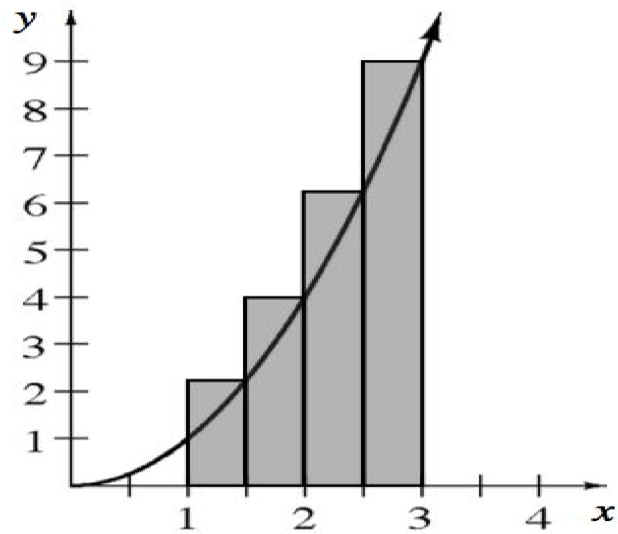
d. Subscript notation will be used to help describe the locations of the rectangles on the interval. Label x_0, x_1, x_2, x_3, and x_4 under each of the corresponding x-values on the graph.

e. The four shaded rectangles in the diagram are called **left endpoint rectangles**. Why are they called left endpoint rectangles?

f. Approximate the area under the curve by writing and evaluating a sum for the total area of the shaded rectangles. Is this sum greater than or less than the actual area under the curve?

#115 Now, take another look at the same situation with a similar, but slightly different approach. The graph of $f(x) = x^2$ for $x \geq 0$ is shown below, but this time the rectangles on the interval $1 \leq x \leq 3$ are different.

a. Label the coordinates of the points where the rectangles touch the curve.



b. In terms of the rectangles drawn, what does the y-value of each coordinate represent?

c. Label x_0, x_1, x_2, x_3, and x_4 under each of the corresponding x-values on the graph.

d. The rectangles drawn are called **right endpoint rectangles**. Why are they called right endpoint rectangles?

e. Approximate the area under the curve by writing and evaluating a sum for the total area of the shaded rectangles. Is this sum greater than or less than the actual area under the curve?

#116

a. Is the graph of $f(x) = x^2$ increasing or decreasing for $1 \leq x \leq 3$?

b. Which type of rectangles produced the underestimate of the actual area under the curve? Why?

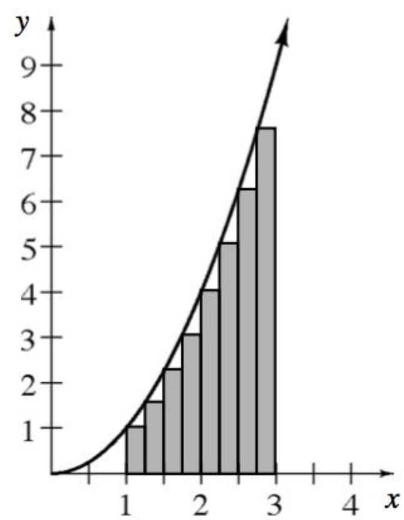
#116 Continued

c. Sketch an example of a curve where the right endpoint rectangles will produce an underestimate of the area under a curve. Is your curve increasing or decreasing?

d. The approximations 6.75 un^2 and 10.75 un^2, from problems 3-114 and 3-115, are not close to one another. In your team, discuss what you can do to make the approximations closer together and record your ideas.

#117 A LITTLE BIT CLOSER NOW

The graph of $f(x) = x^2$ for $x \geq 0$ is shown at below. This time on the interval $1 \leq x \leq 3$ the width of each rectangle has been made smaller.



a. What is the width of each rectangle?

b. What is the height of each rectangle?

c. How are the heights generated from the function?

d. Approximate $A(f, 1 \leq x \leq 3)$. How does this answer compare to the approximations from problems 3-114 and 3-115?

#118	
a. Write a linear equation for x_k. Remember that when $k = 0$ then $x_0 = 1$, and when $k = 4$ then $x_4 = 3$.	b. Write an expression for the heights of the left endpoint rectangles in terms of k.
c. What is the width of each rectangle?	
d. The area of a rectangle is calculated by computing A = (height)(width). Use your expressions from parts (b) and (c) to write an equation for the area of a left endpoint rectangles in terms of k.	
e. Now, use sigma notation and your equation in part (d) to write an expression that can be used to calculate the sum of the areas of the four left endpoint rectangles from problem 3‑114. Remember that for left endpoint rectangles, start with $k = 0$.	

#119 Write the sum for the right endpoint rectangles used in problem 3-115 in sigma notation.
How does the sigma notation for problem 3-115 compare to the sigma notation for problem 3-114? What is the same? What is different?

#120 Write the sum from problem 3-117 in sigma notation for both left endpoint rectangles and right endpoint rectangles.	
Left endpoint rectangles	**Right endpoint rectangles**