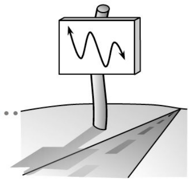
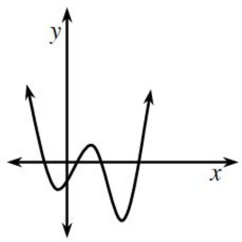
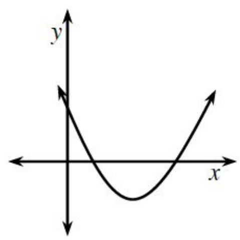
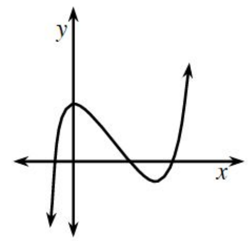
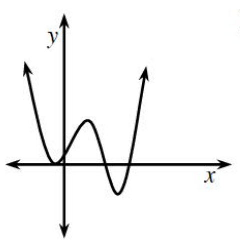
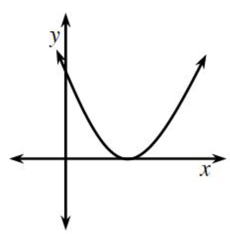
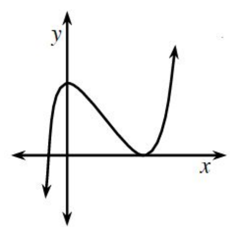
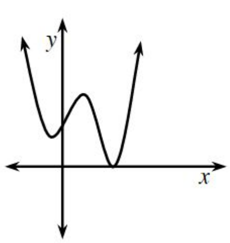
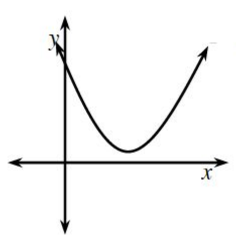
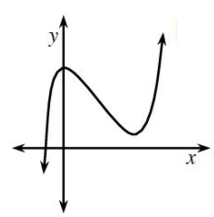
CW#___ Name:___________________________

4.1.3 How can I locate the roots?

Identifying and Using Roots of Polynomials



#35 For each of the following graphs, state the minimum degree of the polynomial and the number of distinct *real* roots. Justify your reasoning.

a.  Minimum degree: Number of distinct real roots:	b.  Minimum degree: Number of distinct real roots:	c.  Minimum degree: Number of distinct real roots:
d.  Minimum degree: Number of distinct real roots:	e.  Minimum degree: Number of distinct real roots:	f.  Minimum degree: Number of distinct real roots:
g.  Minimum degree: Number of distinct real roots:	h.  Minimum degree: Number of distinct real roots:	i.  Minimum degree: Number of distinct real roots:

#36 Use the graphs in problem 4-35, as well as your answers, to complete the parts below.

a. Do the polynomial functions in parts (d) and (g) have fewer roots than the corresponding polynomial in part (a)?

b. Do the polynomial functions in parts (e) and (h) have fewer roots than the corresponding polynomial in part (b)?

c. Do the polynomial functions in parts (f) and (i) have fewer roots than the corresponding polynomial in part (c)?

#37 By the **Fundamental Theorem of Algebra**, a polynomial of degree n has n complex roots. (Recall that every real number is also a complex number.)

a. How many distinct real roots could the polynomial $p(x) = x^3 + 2x^2 + 2x - 5$ have?

b. To determine the real roots of the polynomial, it is helpful to know the factors. What are some possible factors of $p(x)$?

d. How can you determine if a possible factor is actually a factor of the polynomial? Once you decide, work with your team to determine which factor(s) you listed in part (b) is/are factors of $p(x)$. Then write $p(x)$ in completely factored form.

#37 Continued
e. Use your factored form of $p(x)$ from part (c) to determine all of the roots of $p(x)$. Note: Since $p(x)$ is of degree 3, you should determine three roots.
f. How many x-intercepts does the graph of $p(x) = x^3 + 2x^2 + 2x - 5$ have? How many real roots and how many non-real roots does the polynomial have?

Rational Root Theorem

If the polynomial $P(x) = (a_n)x^n + (a_{n-1})x^{(n-1)} + \ldots + (a_1)x^1 + a_0$ has integer coefficients, then every rational zero of P is of the form $\frac{p}{q}$ where p is a factor of the constant term a_0 and q is a factor of the leading coefficient a_n.

Example: $P(x) = 2x^3 + x^2 - 7x + 6$

The possible rational zeros of $P(x)$ are of the form $\frac{\text{factor of 6}}{\text{factor of 2}}$.

The factors of 6 are ±1, ±2, ±3, and ±6. The factors of 2 are ±1 and ±2.
Thus the possible rational zeros are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{3}{2}$, and $\pm\frac{6}{1}$.

#38 Work with your team to determine all of the roots of each of the following polynomial functions. Follow the steps below as you work through each part.

- List all of the possible factors.
- Test possible factors until you find one that works.
- Write the polynomial in completely factored form.
- Use the factors to determine the roots of the polynomial.

a. $p_1(x) = x^4 + 3x^2 - 4$

b. $p_2(x) = 2x^3 - 7x^2 + 5x - 1$

#39 Now work backwards from what you noticed in problem 4-38. For each of the following sets of numbers, write the equation of a polynomial function with integer coefficients that has the given roots.	
a. $-3 + i$ and $-3 - i$	b. $5 + \sqrt{3}$ and $5 - \sqrt{3}$
c. -2 and $\sqrt{7}$	d. 4 and $-3 + i$