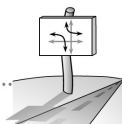


## 4.2.2 Are you still feeling rational?

Graphing Rational Functions

**#64 WHO IS THE GREATEST OF THEM ALL?**

Decide which term of the polynomial has the greatest effect on the graph as we approach positive and negative infinity. How is this related to the end behavior of the function?

a.  $p_1(x) = x^2 + 4x - 12$

b.  $p_2(x) = x^3 - x^2 - 12x + 100$

Examine the polynomials above. Which term of the polynomial has the greatest effect on the graph as we approach positive and negative infinity? How is this related to the end behavior of the function?

**#65** For each function below, substitute in large positive and large negative values of  $x$ . Then decide what happens as  $x \rightarrow \infty$  and also as  $x \rightarrow -\infty$ . Use what you noticed in problem 4-64 to explain your results.

a.  $r_1(x) = \frac{x^2-12}{x^3-x^2}$

b.  $r_2(x) = \frac{x^2+4x-12}{x^3-x^2-12x}$

c.  $r_3(x) = \frac{3x^2+4x-12}{-x^2-12x}$

As  $x \rightarrow \infty$ ,  $r_1 \rightarrow$  \_\_\_\_\_As  $x \rightarrow -\infty$ ,  $r_1 \rightarrow$  \_\_\_\_\_

d.  $r_4(x) = \frac{x^3}{x^2-12}$

e.  $r_5(x) = \frac{5x^3-x^2}{x^3+x^2-12}$

f.  $r_6(x) = \frac{x^3}{-x^2+4x-12}$

**#66** GRAPHING RATIONAL FUNCTIONS, Part 2

$$\text{Let } f(x) = \frac{x^2+5x+6}{x+1} \text{ and } g(x) = \frac{x^2+5x+6}{x+2} .$$

a. For what values of  $x$  is each function undefined? In other words, what values of  $x$  are not in the domain? Explain your reasoning.

b. Use a graphing calculator to make a table and a graph for each function. How are the undefined values in part (a) represented in the table and on the graph for each function?

c. Rewrite each function so that the numerator and denominator are both in factored form.

What do you notice about the function that has the hole? What do you notice about the function that has a vertical asymptote?

**#67** The graph of one of the functions below has a hole, another has vertical asymptotes, and the third has neither. Which has which, and where do the discontinuities exist?

$$f(x) = \frac{x^2-4}{x+2}$$

$$g(x) = \frac{x-6}{(x+3)(x+4)}$$

$$h(x) = \frac{4x}{x^2+9}$$

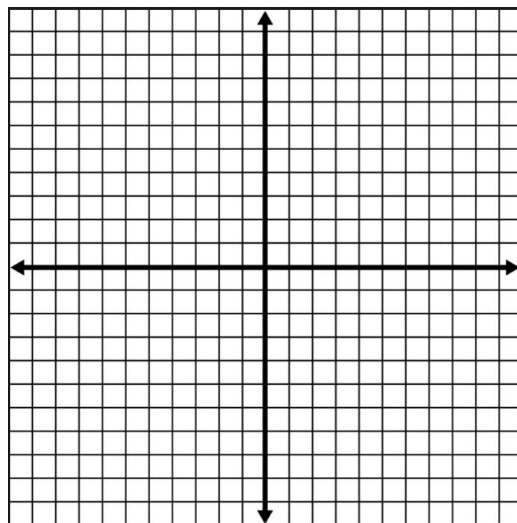
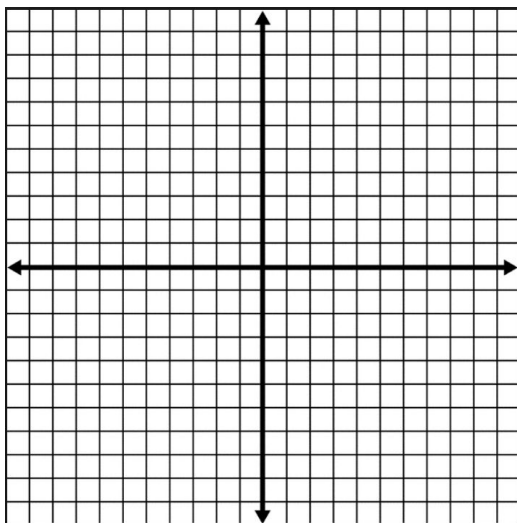
**#69/70** GRAPHING RATIONAL FUNCTIONS, Part 3

a.  $a(x) = \frac{1}{x^2-x-12}$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	

Prediction Graph

Actual Graph



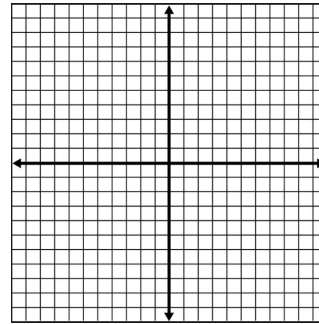
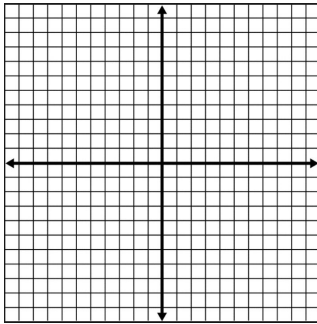
**#69/70 Continued**

b.  $b(x) = \frac{x-4}{x^2-x-12}$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	

Prediction Graph

Actual Graph

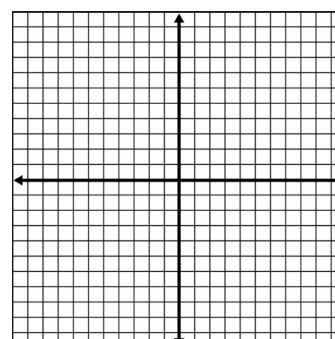
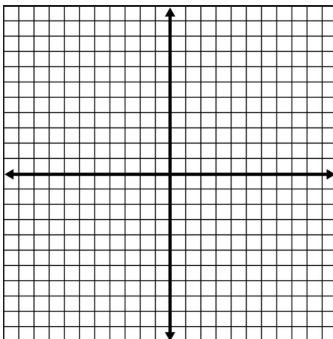


c.  $c(x) = \frac{x^2-16}{x^2-x-12}$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	

Prediction Graph

Actual Graph



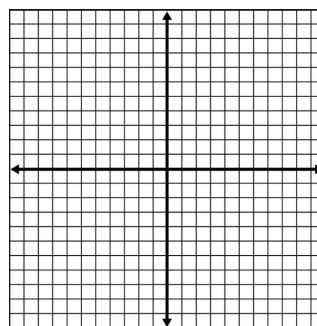
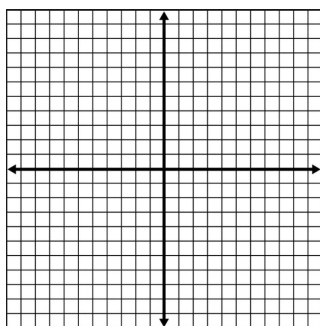
**#69/70 Continued**

d.  $d(x) = \frac{x^2 - x}{x + 3}$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	

Prediction Graph

Actual Graph

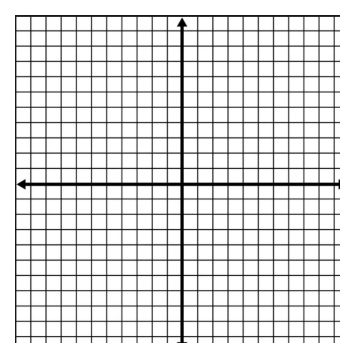
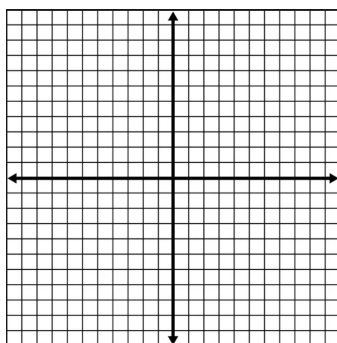


e.  $e(x) = \frac{x^2 - x - 12}{x + 3}$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	

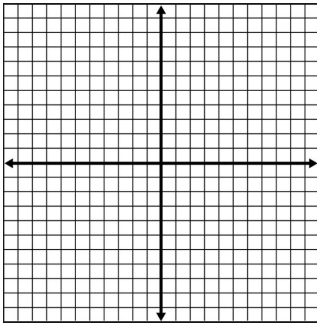
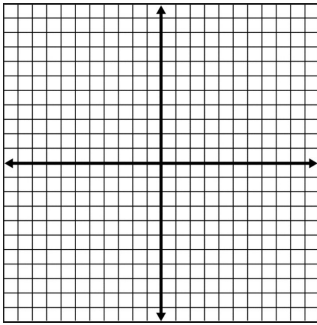
Prediction Graph

Actual Graph



**#69/70 Continued**

$$f. f(x) = \frac{x^2+4x-12}{x+3}$$

Domain	Intercepts	End Behavior	Equation in Simpler Form
		as $x \rightarrow \infty, y \rightarrow \underline{\hspace{2cm}}$  as $x \rightarrow -\infty, y \rightarrow \underline{\hspace{2cm}}$	
Prediction Graph			Actual Graph
			

**#71** Where do the discontinuities come from?

a. Which functions in problem #69 had point discontinuities? What was the same about the equations of these functions?

b. Which functions in problem #69 had vertical asymptotes? What was the same about the equations of these functions?

c. Which functions in problem #69 had horizontal asymptotes? What was the same about the equations of these functions?

d. Which functions in problem #69 had slant asymptotes? What was the same about the equations of these functions?

**#72** What is the equation of the end-behavior function?

a. For the functions in problem #69 that had horizontal asymptotes as their end-behavior functions, write the equation of each horizontal asymptote next to the graph. How can the equation of the asymptote be determined from the equation?

b. A **slant asymptote** occurs when the degree of the polynomial in the numerator is one degree greater than the degree of the polynomial in the denominator. Rewrite the equations of the functions with slant asymptotes using polynomial division. Express any remainder as a fraction.

c. Look at your rewritten equations from part (b). Which term(s) of the equations give the vertical asymptotes of the curves?

d. Given your answer to part (c), which term(s) of the equations give the slant asymptotes of the curves?

e. Predict what the equations of the slant asymptotes are. Then test your predictions with a graphing calculator.

f. What happens when you substitute large positive or negative values of  $x$  into your equations from part (b)? Which term(s) “dominate”? Which become negligible? Why does this make sense with your answers to parts (c) and (d)?