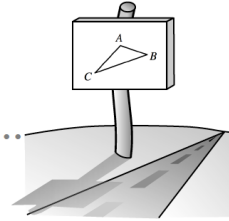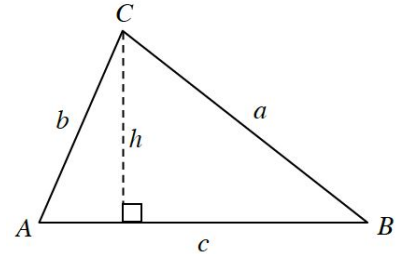**6.1.1** Can I solve a triangle?

The Law of Sines and Area

#2 LAW OF SINES PROOF (ACUTE TRIANGLE)

In the figure at the right, a, b, and c denote the sides opposite angles A, B, and C respectively. Use the diagram to complete the following problems, given that $\triangle ABC$ is an acute triangle.

| a. Write an equation to express h in terms of $\angle A$ and side length b.	b. Write an equation to express h in terms of $\angle B$ and side length a.	c. Use your results from parts (a) and (b) to show that $\dfrac{\sin(A)}{a} = \dfrac{\sin(B)}{b}$.

d. If you drew an altitude to $\overline{BC}$ instead, and followed the same method, what would your result be?

e. Combine results from parts (c) and (d) into an extended proportion (a proportion made up of three equal ratios), to obtain the Law of Sines for an acute triangle.

#4 Nathan lives 200 ft downriver from Tong. Both live across the river from Stacey. Nathan views the other two houses at a 100° angle while Tong sees them at a 38° angle.
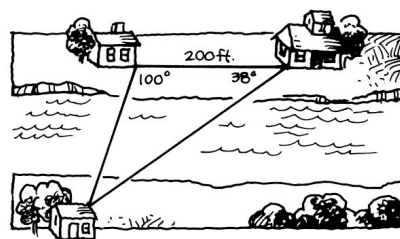

a. Calculate the distance between Nathan and Stacey's houses.

b. Calculate the distance between Tong and Stacey's houses.

#5 A NEW AREA FORMULA

Since side-angle-side determines a unique triangle, it should be possible to calculate its area given the two sides and the angle between those sides. A method similar to the method you used to prove the Law of Sines can be used to generate a formula for the area of a triangle in this situation. Note: Assume $C < 90°$.
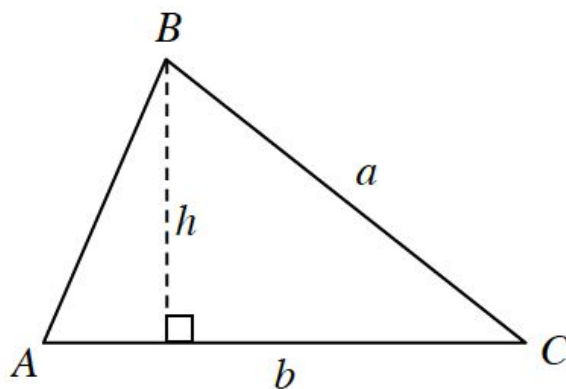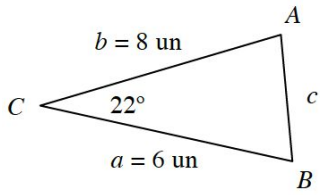

a. Write an equation for the area of $\triangle ABC$.

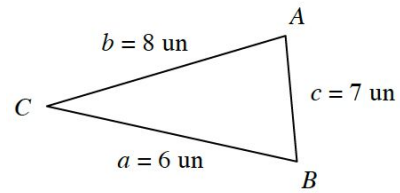b. Using $\angle C$ and hypotenuse a write an expression for the height h.

c. Now combine your results from parts (a) and (b) to write a formula for the area in terms of $\angle C$ and sides a and b.

#7 Solve each triangle below using the Law of Sines and calculate the area using the strategies from this lesson. If you cannot solve a triangle completely, explain why not.
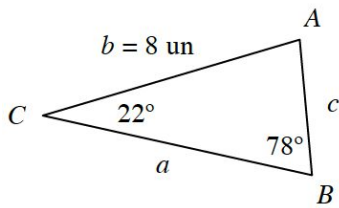
a.

$b = 8$ un
C
$22°$
$a = 6$ un
A
c
B

b.

$b = 8$ un
C
$a = 6$ un
A
$c = 7$ un
B

c.

$b = 8$ un
C
$22°$
a
$78°$
A
c
B

d.

b
C
$22°$
a
$80°$
$78°$
A
c
B