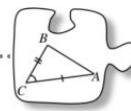


7.2.2 Is there a better way?

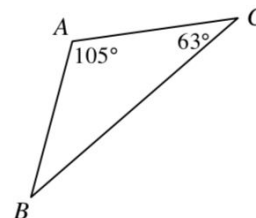


Law of Sines

In problem 7-53, you used a complicated strategy to calculate the side lengths and angle measures of a non-right triangle. Is there a tool you can use to directly calculate the angle measures and side lengths of non-right triangles in fewer steps? Today, you will explore the relationships that exist among the sides and angles of triangles and will develop a new tool called the Law of Sines.

#63.

Is there a relationship between a triangle's side and the angle opposite to it? Based on the angle measures provided in the diagram, which side must be longest? Which side must be shortest? How do you know?



#64.

When Madelyn examined the triangle at right, she says, "I don't think this diagram is drawn to scale because I think the side labeled x has to be longer than 4 cm."



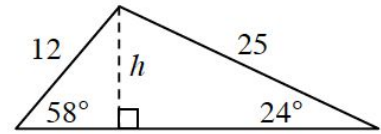
a. Do you agree with Madelyn? Why or why not?

b. Leila thinks that x can be determined by using right triangles. Review what you learned in Lesson 7.2.1 by calculating the value of x .

#65.

Thui and Ivan come up with two different ways to calculate the height of the triangle at right.

- Using the right triangle on the left, Thui writes $\sin(58^\circ) = \frac{h}{12}$.
- Ivan also uses the sine function, but his equation looks like this:
 $\sin(24^\circ) = \frac{h}{25}$.

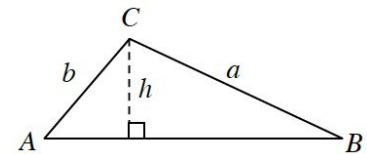


a. Which triangle did Ivan use? Use a colored pencil to trace Ivan's triangle. Then use Ivan's equation to calculate the length of h .

b. Use a different color to trace the triangle that Thui is using. Calculate h using Thui's equation. How do their answers compare?

#66. LAW OF SINES

Edwin wonders if Thui's and Ivan's methods can be used to relate the sides and angles of a non-right triangle. To calculate the height, Ivan and Thui draw a perpendicular segment from vertex C to \overline{AB} . Then, Ivan and Thui each use the sine ratio of a different right triangle.



a. Use the triangle above to write two expressions for h using the individual right triangles as Thui and Ivan did in problem 7-65.

b. Use your expressions from part (a) to show that $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$.

- c. Describe where $\angle B$ is located in relation to the side labeled b . How is $\angle A$ related to the side labeled a ?

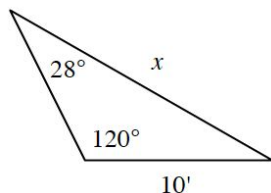
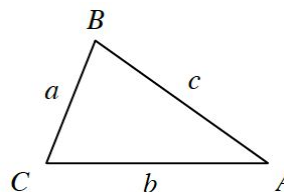
- d. The relationship $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$ is called the Law of Sines. Read the Math Notes box below to learn more about this relationship. Then use this relationship to solve for x in the triangle at below.

Law of Sines

For any $\triangle ABC$, the ratio of the sine of an angle to the length of the side opposite the angle is constant. This means that:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

This property is called the Law of Sines. This is a powerful tool because you can use the sine ratio to solve for the angle measures and side lengths of *any* triangle, not just right triangles. The law works for angle measures between 0° and 180° .



EXTRA PRACTICE:

1. Write the equation of an exponential function of the form $y = ab^x + k$ that passes through $(-1, -0.2)$ and $(4, 2499)$ and has an asymptote at $y = -1$.