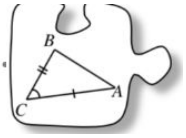
CW#___ Name:_____

Periods: A1 A2 A3 B1 B3

7.2.3 Is there another tool?

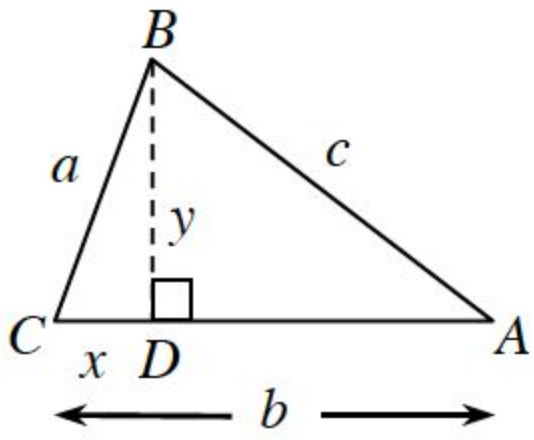


Law of Cosines

#76 LAW OF COSINES

Leila remembers that in problem 7-53 she solved for the missing side length and angle measures of the triangle at right by dividing the triangle into two right triangles. She thinks that using two right triangles may help her figure out a tool that works for any triangle with two given sides and a given angle between them. Help Leila generalize this process by completing the following parts.

a. Examine ΔABC at right. Notice how the side opposite $\angle A$ is labeled a and the side opposite $\angle B$ is labeled b, and so on. Line segment $\overline{BD}$ is drawn so that ΔABC is divided into two right triangles. If $CD = x$, then what is DA?



b. Write an equation relating a, x, and y. Likewise, write an equation relating the side lengths of ΔBDA.

c. Leila notices that both equations from part (b) have a y^2-term. *"Can we combine these equations so that we have one equation that links sides a, b, and c?"* she asks. With your team, use algebra to combine these two equations so that y^2 is eliminated. Then simplify the resulting equation as much as possible.

d. The equation from part (c) still has an x-term. Using only a and $m\angle C$, write an equation relating a, x, and C. Solve your equation for x, then substitute this expression into your equation from part (c) for x.

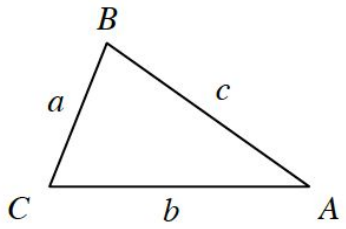
e. Solve your equation from part (d) for c^2. You have now written an equation that relates the lengths of two sides and the measure of the angle between them to the length of the side opposite the included angle. This relationship is called the Law of Cosines.

Law of Cosines

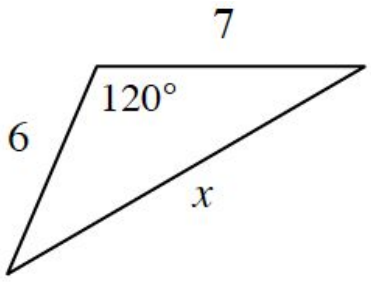
Just like the Law of Sines, the Law of Cosines represents a relationship between the sides and angles of a triangle.

Specifically, when given the lengths of any two sides, such as and , and the angle between them, , the missing information in the triangle can be found using this relationship:

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$

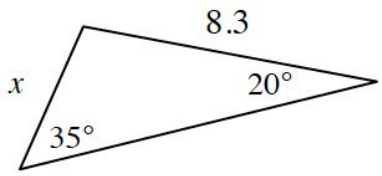


f. Use the Law of Cosines to solve for x in the triangle at right.

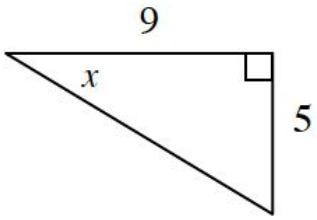


#77 You now have several tools to solve for missing side lengths and angle measures in triangles. Decide which tool to use for each of the triangles below and solve for x. Decide if your answer is reasonable.

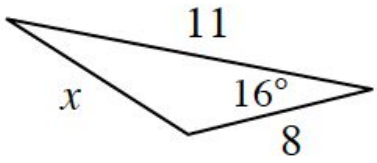
a.



b.



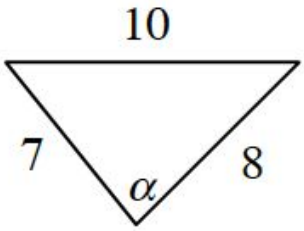
c.



#78 EXTENSION

Not only can the Law of Cosines be used to solve for side lengths, but it can also be used to solve for angle measures. Consider the triangle from Lesson 7.2.1, shown at right.

a. Write an equation that relates the three sides and the angle α. Then solve the equation for α.



b. Now solve for the other two angle measures using any method. Be sure to name which tool(s) you used!