Math 3

G____

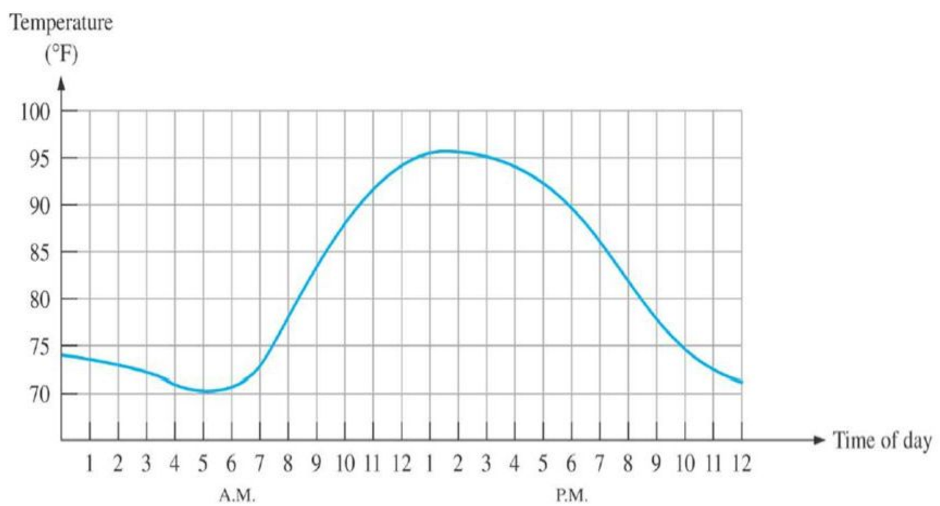
Prerequisites & Investigations Toolkit

Name: ____________________

Period: A1 A2 A3 B1 B3

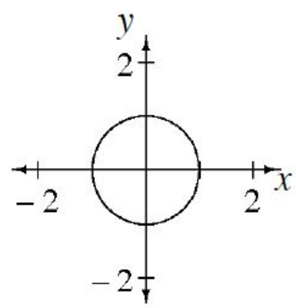
Functions

A relationship between inputs and outputs is a **function** if there is exactly one output (*y*) for each input (*x*).



The example at left is a function because, **for each time of day** (input, on the x-axis) **there is only one temperature** (output, on the y-axis).

It's okay that there are some temperatures (outputs) that occur at more than one time (input).

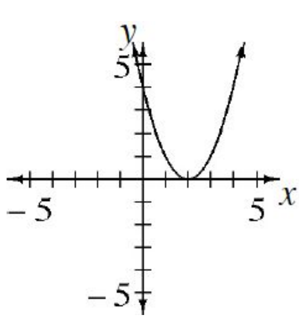


The graph of the circle at right is NOT a function because there are some **x-values that have more than one y-value**. For example, the curve includes the points (0, 1) and (0, -1), so the input of x = 0 has two outputs: y = 1 and y = -1.

The set of possible input values for a function is called the **domain**. This set contains every *x*-value for which the function is defined.

The **range** of a function is the set of possible output values. This set contains every *y*-value that the function can generate.

$y = (x - 2)^2$



This parabola has a **domain** of all real numbers because any value of x will work in the equation, any value of x will have a point on the curve. This can be written with numbers and symbols as:

Domain: **Inequality Notation** **Interval Notation**

Since the parabola has its vertex at (2, 0), we know that y = 0 is the lowest y-value. Therefore, the **range** is all real numbers that are greater than or equal to zero. This can be written with numbers and symbols as:

Range: **Inequality Notation** **Interval Notation**

Sometimes you can tell the domain and range of a function without looking at the graph. For example,

$$y = \sqrt{x} + 5$$

The function above has a **restricted domain** because we can't take the square root of negative numbers. Therefore, the domain will be:

Domain: **<u>Inequality Notation</u>** **<u>Interval Notation</u>**

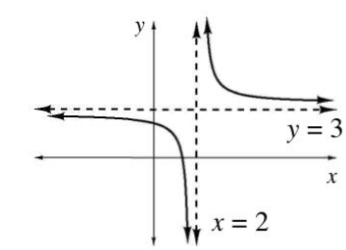
The restricted domain also affects the range. Since the lowest x-value within the domain is x = 0 the lowest y-value will be y = 5. Therefore the range will be:

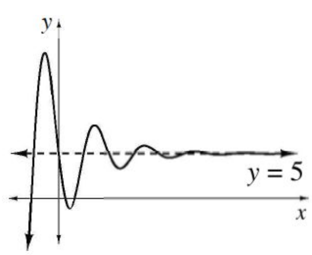
Range: **<u>Inequality Notation</u>** **<u>Interval Notation</u>**

Graphs with Asymptotes

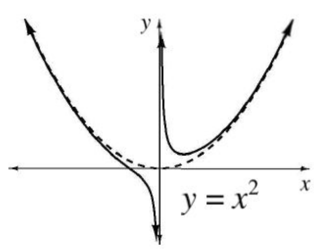
A mathematically clear and complete definition of an asymptote requires some ideas from calculus, but some examples of graphs with **asymptotes** should help you recognize them when they occur. In the following examples, the dotted lines are the asymptotes, and the equations of the asymptotes are given. In the two lower graphs, the *y*-axis, *x* = 0, is also an asymptote.

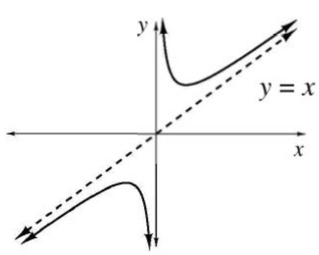
As you can see in the examples, asymptotes can be diagonal lines or even curves.





Draw an example of an exponential function





In this course, asymptotes will almost always be horizontal or vertical lines. The graph of a function has a horizontal asymptote if, as you trace along the graph out to the left or right (that is, as you choose *x*-values farther and farther away from zero), the distance between the graph of the function and the asymptote gets smaller and smaller.

A graph has a vertical asymptote if, as you choose *x*-coordinates closer and closer to a certain value, from either the left or right (or both), the *y*-coordinate gets farther away from zero.

Polynomials

A **polynomial** expression in one variable is an expression that can be written as the sum or difference of terms of the form:

$$(\text{any real number})x^{(\text{whole number})}$$

Polynomials in one variable (often x) are usually arranged with powers of x in order, starting with the highest, from left to right. For example: $x^8 - x^5 + x^2 - x + 4$

The highest power (______________________) of the variable in a polynomial in one variable is called the ___________________ **of the polynomial**. The numbers that are multiplied by the variable in each term are called _______________________..

Polynomials have special names depending on how many terms they have. A polynomial with just 1 term is called a ____________________. If there are2 terms, it's called a ___________________. If it has 3 terms, it's called a ____________________. If there are more than three terms, it doesn't have a special name.

Example 1: $$3x + 5$$	This is a _________________ of degree ____ with ____ terms. The coefficients are ___ and ___. We can think of 5 as a coefficient if we imagine there term is $5x^0$, since $x^0 = 1$. The term without a variable, ____ is also called the **constant term**.
Example 2: $$7x^5 + 2.5x^3 - ½\, x + 4$$	This is a _________________ of degree ___ with coefficients 7, _____, _____, and 4. The constant term is ____. In this expression, 7 is called the _____________ ________________ because it is the coefficient of the term with the highest power.
Example 3: $2(x + 2)(x + 5)$ is a polynomial in factored form with degree 2 because it can be written in standard form as $$2x^2 + 14x + 20$$	In standard form, this is a __________________ . It has coefficients _____, _____ and _____. The related quadratic function $f(x) = 2x^2 + 14x + 20$ is a polynomial function of degree ____.

Laws of Exponents

In the expression x^3, x is the **base** and 3 is the **exponent**. Here are the **laws of exponents** with examples:

Law	Examples	
$x^m x^n = x^{m+n}$ for all x	$x^3 x^4 = x^{3+4} = x^7$	$2^5 \cdot 2^{-1} = 2^4$
$\frac{x^m}{x^n} = x^{m-n}$ for $x \neq 0$	$x^{10} \div x^4 = x^{10-4} = x^6$	$\frac{5^4}{5^7} = 5^{-3}$
$(x^m)^n = x^{mn}$ for all x	$(x^4)^3 = x^{4 \cdot 3} = x^{12}$	$(10^5)^6 = 10^{30}$
$x^0 = 1$ for $x \neq 0$	$\frac{y^2}{y^2} = y^0 = 1$	$9^0 = 1$
$x^{-n} = \frac{1}{x^n}$ for $x \neq 0$	$x^{-2} = \frac{1}{x^2}$	$3^{-1} = \frac{1}{3}$
$x^{m/n} = \sqrt[n]{x^m}$ for $n \neq 0$	$x^{4/5} = \sqrt[5]{x^4}$	$8^{2/3} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$

Describing Graphs

When you are asked to **describe a graph**, ask yourself the following characteristics:

- **Shape:** linear, nonlinear, quadratic, cubic, etc.
- **Line of symmetry:** such as with parabolas, absolute value functions, inverses, etc. Label with its equation.
- **Asymptotes:** (see above) such as with rational functions (hyperbolas) and exponential functions. Label with its equation.
- **Orientation:** parabolas and absolute value functions are oriented positively when they open upwards.
- **Increasing or decreasing:** as the values of x increase, is the curve (or line) going up or down?
- ***x*- and *y*-intercepts:** if relevant, explain what these mean in context.
- **Other Important points:** such as the **vertex** of a parabola, the **endpoint** of a square root function, or **maximum** and **minimum** points.
- **Domain & range:** possible values of x and y, respectively. If relevant, explain why this makes sense in context.