Precalculus Honors

Expressions and Equations Toolkit

Name: ______________________

Date: ______________________

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Operations with Rational Expressions

Operations with rational expressions are completed using the same methods used to complete operations with numeric fractions. Review the examples below.

To multiply or divide rational expressions, both the numerator and denominator need to be in factored form. To add or subtract, only the denominators need to be in factored form.

Multiplication: $\frac{x^2+6x}{(x+6)^2} \cdot \frac{x^2+7x+6}{x^2-1}$ for $x \neq -6, -1$, or 1

Factor: $\frac{x(x+6)}{(x+6)(x+6)} \cdot \frac{(x+1)(x+6)}{(x+1)(x-1)}$

Reorder the factors: $\frac{(x+6)}{(x+6)} \cdot \frac{(x+6)}{(x+6)} \cdot \frac{x}{(x-1)} \cdot \frac{(x+1)}{(x+1)}$

Since $\frac{(x+6)}{(x+6)} = 1$ and $\frac{(x+6)}{(x+6)} = 1$, simplify: $1 \cdot 1 \cdot \frac{x}{(x-1)} \cdot 1 \Rightarrow \frac{x}{(x-1)}$

Note: Division is carried out using the same process, but first rewrite the division problem as a multiplication problem.

Addition: $\frac{4}{x^2+5x+6} + \frac{2x}{x+2}$ for $x \neq -3, -2$

Factor: $\frac{4}{(x+2)(x+3)} + \frac{2x}{(x+2)}$

Create a common denominator: $\frac{4}{(x+2)(x+3)} + \frac{2x}{(x+2)} \cdot \frac{(x+3)}{(x+3)}$

Complete the addition: $\frac{4+2x(x+3)}{(x+2)(x+3)}$

Rewrite: $\frac{4+2x^2+6x}{(x+2)(x+3)}$ or $\frac{2x^2+6x+4}{(x+2)(x+3)}$

Keep simplifying (if possible): $\frac{2(x^2+3x+2)}{(x+2)(x+3)} = \frac{2(x+2)(x+1)}{(x+2)(x+3)} = \frac{2(x+1)}{(x+3)}$

Note: Subtraction is carried out using the same process.

Answers may be written with or without parentheses.

Simplifying Complex Fractions

A **complex fraction** is fraction in which the numerator and/or denominator also contain fractions. Two methods for rewriting a complex fraction are shown below.

Example: $\frac{\frac{a}{b}}{1-\frac{1}{c}}$

Method 1: Multiply numerator and denominator by ***bc*** (common denominator):

$$\frac{\left(\frac{a}{b}\right)}{\left(1-\frac{1}{c}\right)} \cdot \frac{\frac{bc}{1}}{\frac{bc}{1}} \quad \frac{\frac{abc}{b}}{bc-\frac{bc}{c}}$$

Now simplify: $\frac{\frac{abc}{b}}{bc-\frac{bc}{c}} = \frac{ac}{bc-b}$

Method 2: Multiply the denominator by $\frac{c}{c}$ to create division of fractions:

$$\frac{\frac{a}{b}}{\left(1-\frac{1}{c}\right)\frac{c}{c}} = \frac{\frac{a}{b}}{\frac{c-1}{c}}.$$

Then divide the fractions: $\frac{a}{b} \div \frac{c-1}{c} = \frac{a}{b} \cdot \frac{c}{c-1} = \frac{ac}{bc-b}$

Polynomial Division with Remainder

The examples below show two methods for dividing. Both have remainders, which are written as fractions.

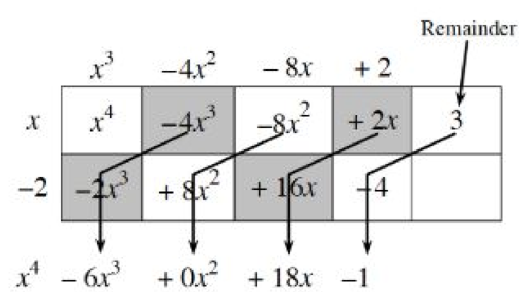
$$\frac{x^4-6x^3+18x-1}{x-2}$$

Using **long division**:

$$\begin{array}{r} x^3-4x^2-8x+2 \\ x-2\,\overline{)\,x^4-6x^3+0x^2+18x-1} \\ -\underline{(x^4-2x^3)} \\ -4x^3+0x^2 \\ -\underline{(-4x^3+8x^2)} \\ -8x^2+18x \\ -\underline{(-8x^2+16x)} \\ 2x-1 \\ -\underline{(2x-4)} \\ 3 \leftarrow \text{Remainder} \end{array}$$

Final answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$

Using an **area model**:



Final answer: $x^3-4x^2-8x+2+\frac{3}{x-2}$