Precalculus Honors

Functions Toolkit

Name: ______________________

Date: ______________________

Period:____

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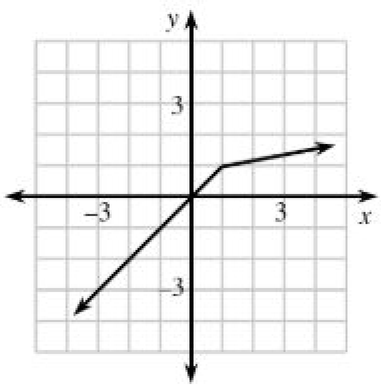
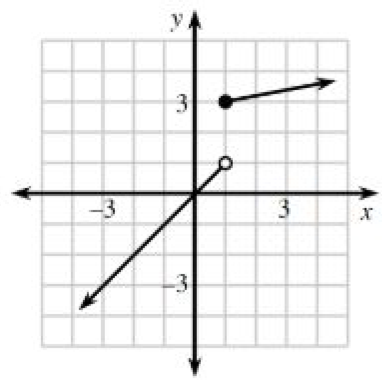
The Intuitive Notion of Continuity

The formal definition of continuity will be given later in the course. Intuitively, a function is **continuous** if the graph of the function can be drawn without lifting your pencil from the paper. Try tracing each of the curves below to understand this idea.

Continuous



Not Continuous



Piecewise-Defined Functions

A **piecewise-defined function** is defined as a function that is composed of two or more functions. Each part usually consists of a function with a restricted domain.

Example:

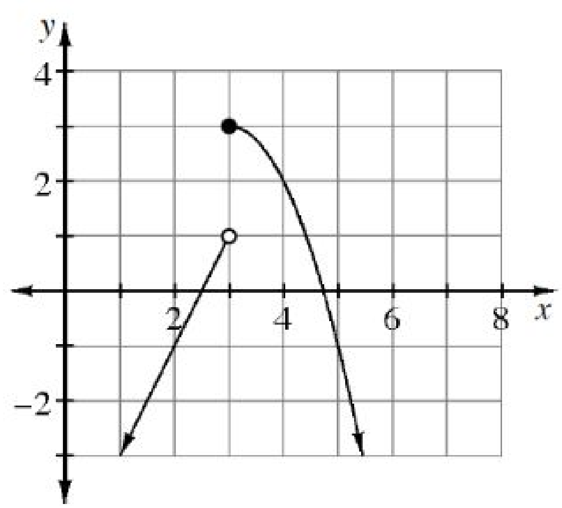
$$f(x) = \begin{cases} 2x - 5 & \text{for } x < 3 \\ -(x-3)^2 + 3 & \text{for } x \geq 3 \end{cases}$$

For $x < 3$

x	$2x - 5$
0	–5
1	–3
2	–1
3	1

For $x \geq 3$

x	$-(x-3)^2 + 3$
3	3
4	2
5	–1
6	–6



Notice that the "break" in the function is at $x = 3$. $x = 3$ is included in both sections of the table. One section of the graph has an open circle at $x = 3$, while the other section has a closed circle.

Describing Functions

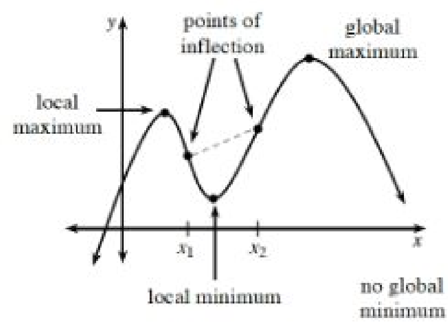
The formal definitions of an increasing or decreasing function, are as follows:

A function, f, is **increasing** on an interval if, for all $x_1 < x_2, f(x_1) < f(x_2)$.

A function, f, is **decreasing** on an interval if, for all with $x_1 < x_2, f(x_1) > f(x_2)$.

Maximum and minimum points are collectively called **extremum** (plural **extrema**). Intuitively, a **maximum** point (plural **maxima**) occurs where the graph of a function changes from increasing to decreasing. This is a **global maximum** if it is the highest point for the entire function; otherwise it is a **local maximum**.

Likewise, a **minimum** point (plural **minima**) occurs where the graph of a function changes from decreasing to increasing. This is a **global minimum** if it is the lowest point for the entire function; otherwise it is a **local minimum**.

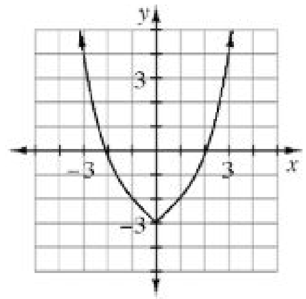


Intuitively, a graph is **concave up** over an interval $x_1 < x < x_2$ if a line segment joining any two consecutive inflection points on the graph over that interval lies completely above the graph. A graph is **concave down** over an interval $x_1 < x < x_2$ if a line segment joining any two consecutive inflection points on the graph over that interval lies completely below the graph. The **point of inflection** is the point at which the concavity changes.

Even and Odd Functions

When $f(-x) = f(x)$, the function f is called an **even function**.
For example, given $f(x) = |x| + 0.05x^4 - 3$:

$$\begin{aligned} f(-x) &= |-x| + 0.05(-x)^4 - 3 \\ &= |x| + 0.05x^4 - 3 \\ &= f(x) \end{aligned}$$

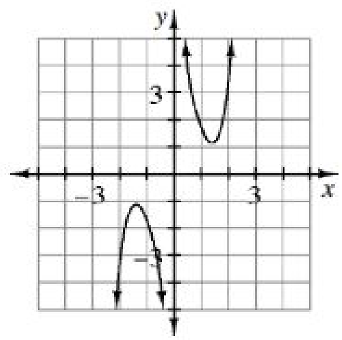


Therefore, $f(x) = |x| + 0.05x^4 - 3$ is an even function.

The graph of an even function has reflective symmetry across the y-axis.

When $f(-x) = -f(x)$, the function f is called an **odd function**.
For example, given $f(x) = \frac{2}{x} - 0.5x^3 + 0.2x^5$:

$$\begin{aligned} f(-x) &= \frac{2}{(-x)} - 0.5(-x)^3 + 0.2(-x)^5 \\ &= -\frac{2}{x} + 0.5x^3 - 0.2x^5 \\ &= -f(x) \end{aligned}$$



Therefore, $f(x) = \frac{2}{x} - 0.5x^3 + 0.2x^5$ is an odd function.

The graph of an odd function has 180° rotational symmetry about the origin.

Transformations of Functions and Graphing Form

If $y = f(x)$ is a function, then the equation in **graphing form** transformations of this function can be written as:

$$y = a \cdot f(b(x - h)) + k$$

(h, k) is the **locator point** because it helps us to locate the transformed graph.

The parameters transform the parent graph as follows:

- horizontally translated by h units
- vertically translated by k units
- vertically stretched/compressed by a factor of a
- horizontally stretched/compressed by a factor of b
- reflected vertically if $a < 0$
- reflected horizontally if $b < 0$