Math 3

Name: ______________________

TOOLKIT #6:
Inverses
(Chapter 5)

Date: ______________________

G_

Period: A1 A2 A3 B1 B2 B3

What is an inverse function?	
Formal Definition: A function that "undoes" what the original function does. The inverse function performs in reverse order the inverse operations of the function	**In my own words:**

Inverse Operations:			
What is the inverse of....?	**Addition** Ex. +2	subtract or add the opposite (-) Ex. -2 Ex. $+(-2)$	
	Multiplication Ex. ×10	divide or multiply by reciprocal Ex. $\div 10$ Ex. $\cdot \frac{1}{10}$	
	Square (exponent) Ex. $(\ )^2$	Square root or exponent of $\frac{1}{2}$ Ex. $\sqrt{\ }$ Ex. $(\ )^{\frac{1}{2}}$	
	Cube (exponent) Ex. $(\ )^3$	Cube root or exponent of $\frac{1}{3}$ Ex. $\sqrt[3]{\ }$ Ex. $(\ )^{\frac{1}{3}}$	
	***x* is the exponent** Ex. 2^x	¯_(ツ)_/¯ We'll learn about this next unit...	
	Absolute Value Ex. $\lvert x \rvert$	Maybe ± x, but a with domain restriction...check the graph!	

G.E.M.A.

"G" stands for Grouping Symbols, including parentheses ( ), brackets [ ], radicals √, fraction bars —, absolute value symbols | |, etc.

"E" stands for Exponents (and Roots!). These can be considered the same because any root could be written as an exponent instead. For example... $\sqrt{x} = x^{\frac{1}{2}}$

"M" stands for Multiplication(and Division!). These can be considered the same because division is the same as multiplying by the reciprocal. For example... $x \div 8 = x \cdot \frac{1}{8}$

"A" stands for Addition(and Subtraction!). These can be considered the same because subtraction is the same as adding the opposite. For example... $x - 9 = x + (-9)$

Do/Undo Tables (Writing the equation of an inverse STRATEGY #1)

A do/undo table can help you figure out how to write the equation of an inverse.

STEP 1: Ask yourself, "what operations does this equation do to the input, x?" Record those operations on the **Do** ↓ side of the table.

STEP 2: On the **Undo** ↑ side, record the inverse operation for each operation of the function.

STEP 3: Write an equation to represent the inverse operations in the reverse order.

STEP 4: Simplify the inverse equation if necessary.

$f^{-1}(x) = \sqrt[3]{2(x-5)} + 4$

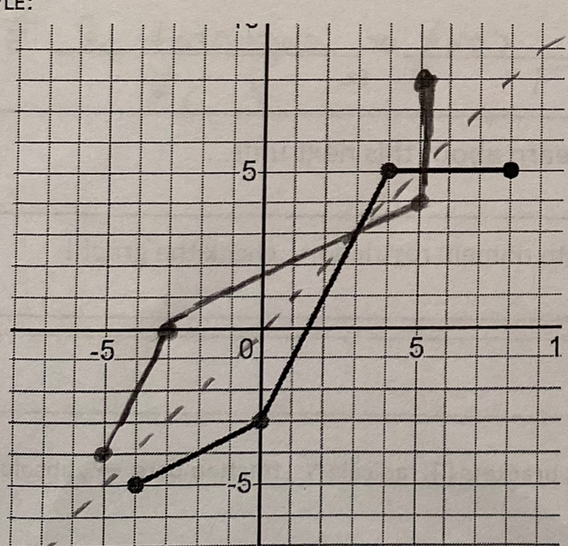
Ex. $f(x) = \frac{1}{2}(x-4)^3 + 5$

Do ↓	Undo ↑
-4	$+4$
$(\ )^3$	$\sqrt[3]{\ }$
$\cdot \frac{1}{2}$	$\cdot 2$
$+5$	-5

Multiple Representations of Inverses

Graphs:
The graphs of inverses will have reflective symmetry. The line of symmetry is always $y=x$.

EXAMPLE:



Tables:
The tables of inverses are related because the x & y values switch.

EXAMPLE:

Original f(x)

x	y
–3	6
–2	0
–1	–4
0	–6
1	–6
2	–4
3	0
4	6

Inverse $f^{-1}(x)$

x	y
6	-3
0	-2
-4	-1
-6	0
-6	1
-4	2
0	3
6	4

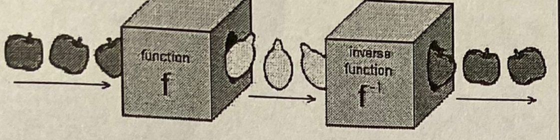
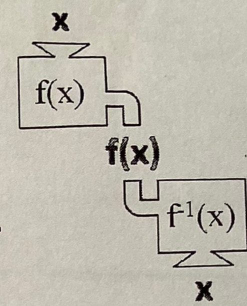
Domain & Range:
The Domain & Range of an equation and its inverse are related because They switch.

EXAMPLE:

Original g(x):
Domain: $-4 \leq x < \infty$
Range: $\infty < y \leq 18$

Inverse $g^{-1}(x)$:
Domain: $\infty < x \leq 18$
Range: $-4 \leq y < \infty$

How can you check that an inverse equation is correct?

Method #1:	Method #2:
Choose a value of x to test... 	Composition of Functions... To generalize the process in Method #1, the input is x... 
EX. $f(x) = -2x^3 + 5$ and $f^{-1}(x) = \sqrt[3]{\frac{x-5}{-2}}$	EX. $f(x) = -2x^3 + 5$ and $f^{-1}(x) = \sqrt[3]{\frac{x-5}{-2}}$
If the original input is 2, $f(2) = -2(2)^3 + 5$ $= -2(8) + 5$ $= -16 + 5$ $= -11$	If the original input is x, the output is $-2x^3 + 5$. If the input for the inverse is $-2x^3 + 5$, $f^{-1}(f(x)) = \sqrt[3]{\frac{(-2x^3+5)-5}{-2}}$ (simplify) $= \sqrt[3]{\frac{-2x^3}{-2}} = \sqrt[3]{x^3} = x$ ✓
Now, the output of f will be the input of f^{-1}... $f^{-1}(-11) = \sqrt[3]{\frac{-11-5}{-2}} = \sqrt[3]{\frac{-16}{-2}}$ $= \sqrt[3]{8}$ $= 2$	
The output of $f^{-1}(x)$ is equal to the original input, so this example supports the claim that these two equations are inverses.	The output of $f^{-1}(f(x)) = x$, the original input. This proves that f^{-1} will always undo f. To prove they are inverses, you must also show that $f(f^{-1}(x)) = x$ also.

Function Notation for Inverses
The symbol $f^{-1}(x)$ is read "f inverse of x" and it is used to say that this function is the inverse of $f(x)$

x-y Interchange (Writing the equation of an inverse STRATEGY #2)	
STEP 1: Write the original equation with y instead of function notation. Rewrite the equation with x and y switched.	Ex. $h(x) = \frac{x}{x+2}$ $\quad h^{-1}(x) = \frac{2x}{1-x}$ $y = \frac{x}{x+2}$
STEP 2: Solve for y.	$(y+2)(x) = \left(\frac{y}{y+2}\right)(y+2)$ $xy + 2x = y$ $2x = y - xy$ $\frac{2x}{1-x} = \frac{y(1-x)}{1-x}$ $\frac{2x}{1-x} = y$