Period: B1

Transformations Toolkit

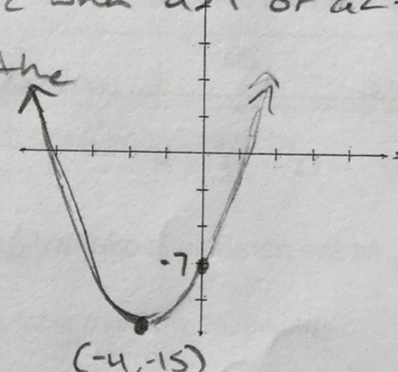
Quadratic Functions

Standard Form: $y = ax^2 + bx + c$, for which $a \neq 0$

This form of the equation shows 3 important characteristics of the graph:

1. The sign (+/-) of a determines The orientation (opens up or down)
2. The value of a is vertical stretch or compression factor
   when $-1<a<1$ it compresses & when $a\geq 1$ or $a\leq -1$ it stre
3. The value of c is The constant & also the y-intercept.

Example: $y = \frac{1}{2}x^2 + 4x - 7$



For any y-intercept, the x-value is equal to zero

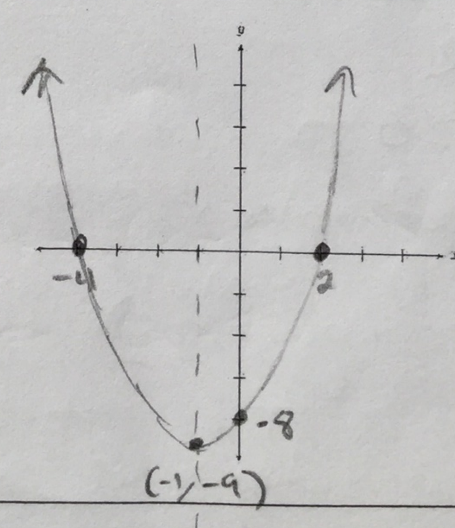
Factored Form: $y = a(x-m)(x-n)$, for which $a \neq 0$

This form of the equation shows 3 important characteristics of the graph:

1. The sign (+/-) of a determines the orientation (up or down)
2. The value of a is the stretch or compression factor
3. The value of m and n are the x-intercepts

(-3) (3)

Example: $y = (x-2)(x+4)$



For any x-intercept, the y-value equals zero

Graphing Form: $y = a(x-h)^2 + k$, for which $a \neq 0$

This form of the equation shows 4 important characteristics of the graph:

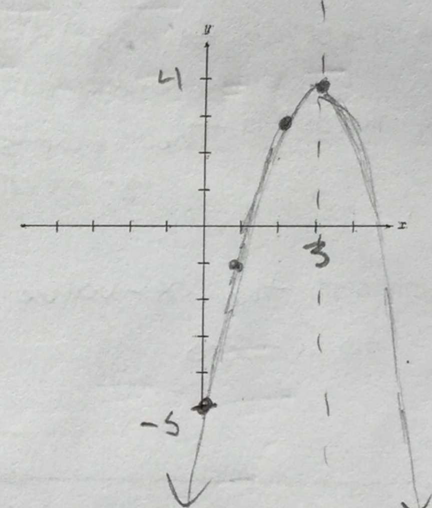
1. The sign (+/-) of a determines the orientation (up or down)
2. The value of a is the stretch factor
3. The value of h determines the horizontal shift
4. The value of k determines the vertical shift

→ This means (h, k) is the vertex

→ and $x = h$ is the

line of symmetry.

Example: $y = -(x-3)^2 + 4$



a is (−), so the parabola is oriented down and

The vertex is (3,4)

The Parent Function: $y = x^2$

Vertex: (0,0)

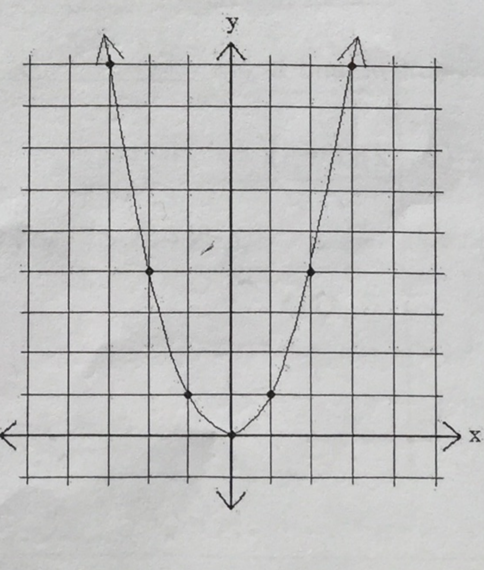
Line of Symmetry: x = 0

Orientation: up

Domain: ℝ

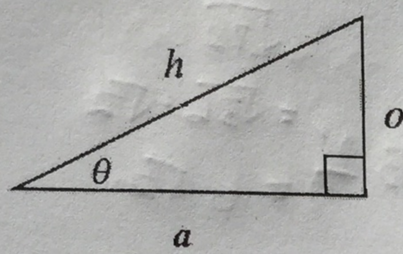
Range: y ≥ 0

Shape: parabola



Trigonometric Ratios

There are three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any **right triangle**. In the triangle below, when the sides are described relative to the angle θ, the opposite leg is y and the adjacent leg is x. The hypotenuse is h regardless of which acute angle is used.



$\tan\theta = \frac{opp}{adj} = \frac{o}{a}$

$\sin\theta = \frac{opp}{hyp} = \frac{o}{h}$

$\cos\theta = \frac{adj}{hyp} = \frac{a}{h}$

Soh Cah Toa

Exponential Models

Situations that grow by equal factors (multipliers) over equal intervals can often be modeled by exponential functions. An **exponential function** is a function of the form:

$f(x) = a \cdot b^x$, *where* $b > 0$, $b \neq 0$, *and* $a \neq 0$.

Note that the variable x is the exponent. All exponential functions of this form have the following characteristics:

→ There is a horizontal asymptote on the x-axis because $f(x) \neq 0$

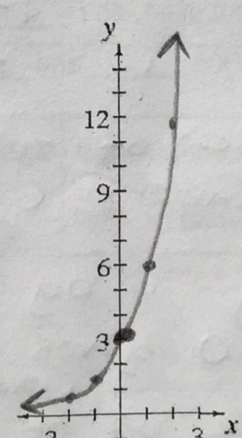
→ a is the value of the function when $x = 0$, or the y-intercept because $b^0 = 1$ and $1 \cdot a = a$

→ b is the base, also called the multiplier.

For example,

$y = 3 \cdot 2^x$ has parameters $a =$ 3 and $b =$ 2.

x	y	
−2	.75	
		× 2
−1	1.5	
		× 2
0	3	
		× 2
1	6	
		× 2
2	12	



Rewriting Radical Expressions

Before calculators were readily available, people found it convenient to rewrite square root expressions in a simplified form to make it easier to perform calculations, combine, and compare them. A square root is simplified when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Example:

$\sqrt{72}$ can be rewritten as $\sqrt{36} \cdot \sqrt{2}$, or as $\sqrt{9} \cdot \sqrt{8}$ and you still get the same result when you simplify completely.

$\sqrt{72}$
$= \sqrt{9} \cdot \sqrt{8}$
$= \sqrt{9} \cdot \sqrt{4} \cdot \sqrt{2}$
$= 3 \cdot 2 \cdot \sqrt{2}$
$= 6\sqrt{2}$

or

$\sqrt{72}$
$= \sqrt{36} \cdot \sqrt{2}$
$= 6\sqrt{2}$

Verify with your calculator that both $6\sqrt{2}$ and $\sqrt{72} \approx 8.485$.

Example: $\sqrt{72} + \sqrt{18}$

To add terms with radicals, the radicals need to be alike. Verify your simplification with your calculator.

$\sqrt{72} + \sqrt{18}$
$= 6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

It is difficult to estimate the value of a number with a radical in the denominator, and it is also difficult to combine it with or compare it to other numbers. For these reasons, it is sometimes helpful to **rationalize the denominator** so that no radical remains in the denominator.

Example: $\frac{10}{\sqrt{2}}$

First, multiply the numerator and denominator by the radical in the denominator. Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the expression.

After multiplying, notice that the denominator no longer has a radical, since $\sqrt{2} \cdot \sqrt{2} = 2$.

Often, the product can be further simplified.

$\frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{10\sqrt{2}}{2}$
$= 5\sqrt{2}$

Imaginary and Complex Numbers

The imaginary number i is defined as the square root of –1, so $i =$ $\sqrt{-1}$. Therefore $i^2 = -1$, and the two solutions of the equation $x^2 + 1 = 0$ are $x =$ i and $-i$.

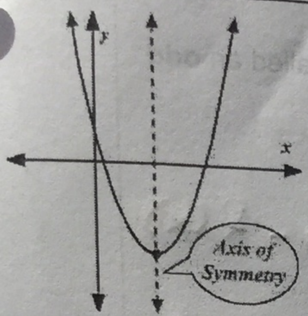
Real Numbers: The collection (set) of zero and all positive and negative numbers (integers, decimals, irratiand #s, fractions)

Imaginary Numbers: Multiplying i by every possible real number excluding 0 gives us this set of numbers (example: 2i, 3i...)

Complex Numbers: The set made up of the sum of a real number and an imaginary number such as 2+i written in the form a+bi (a & b are real #s)

eal are complex if b=0
& imaginary are complex if a=0

Determining the Vertex of a Parabola



Averaging the x-Intercepts:

Due to the symmetry of parabolas, the x-value of the **vertex** will always be the midpoint between the x-intercepts. The vertex of a parabola always lies on the **axis of symmetry.**

Example:

First, determine the x-intercepts.

$y = (x-5)(x+4)$

(5,0) (-4,0)

Next, average the x-values of the intercepts to determine the x-value of the vertex.

$\frac{5+-4}{2} = \frac{1}{2}$

Then, substitute the x-value of the vertex in the equation so that you can solve for y.

$y = (.5-5)(.5+4)$

$= (-4.5)(4.5)$

$= -20.25$

Therefore, the vertex of $y = (x-5)(x+4)$ is

(.5, 20.25)

or

$(\frac{1}{2}, -20\frac{1}{4})$

Point-Slope Equations for Lines

If you think of $y = x$ as a **parent function**, then the **graphing form** for the family of linear functions can be written as:

$$y = a(x-h) + k$$

When the equation of a linear function is in the form $y - k = a(x-h)$ it is often called the **point-slope form** of the equation for a line that contains the point (h, k) and has slope a.

For example, if you know a line contains the point (7, –8) and has slope –4, then the equation of the line can be written as:

$y - (-8) = -4(x-7)$

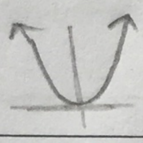
$y + 8 = -4(x-7)$

Even and Odd Functions

Even Functions:
When $f(-x) = f(x)$, the function f is called an **even function.**

For example, given $f(x) = x^2$:

$f(-x) = (-x)^2 = x^2 = f(x)$

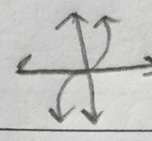


symmetry across y-axis

Odd Functions:
When $f(-x) = -f(x)$, the function f is called an **odd function.**

For example, given $f(x) = x^3$:

$f(-x) = (-x)^3 = -x^3 = -f(x)$



rotational symmetry about origin

Equations in Graphing Form for Families

If $y = f(x)$ is an equation for a parent graph, then the equation in graphing form for the family of functions with similar characteristics as $f(x)$ can be written as:

$$y = a \cdot f(x - h) + k$$

We call (h, k) the **locator point** because it helps us to locate the transformed graph. The parameters transform the parent graph as follows:

- **horizontally translated** by the value of h
- **vertically translated** by the value of h
- **vertically stretched** if the absolute value of $|a| > 1$
- **vertically compressed** if the absolute value of $|a| < 1$
- **reflected** across the x-axis if $-a$

You should be familiar with the following families of functions:

Parent	Family	Graphing Form
$y = x$	Linear	$y = a(x-h) + h$
$y = \lvert x \rvert$	Absolute Value	$y = a\lvert x-h \rvert + h$
$y = x^2$	Quadratic (Parabola)	$y = a(x-h)^2 + h$
$y = x^3$	Cubic	$y = a(x-h)^3 + k$
$y = \frac{1}{x}$	Rational (Hyperbola)	$y = a\left(\frac{1}{x-h}\right) + k$
$y = \sqrt{x}$	Square Root	$y = a\sqrt{x-h} + k$
$y = b^x$	Exponential	$y = a \cdot b^{(x-h)} + h$