Precalculus Honors

Sigma Notation Toolkit

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Sigma Notation

The capital Greek letter sigma, $\sum$, (equivalent to the capital "S" in English) is used in mathematics as a compact way to indicate sums. Use **sigma notation** (also called summation notation) as a shorter way to write a long list of numbers or terms being added together.

Example: $\sum_{n=1}^{3} n^2 = 1^2 + 2^2 + 3^2 = 14$ means the sum of the expression n^2 evaluated for $n = 1$, 2, and 3.

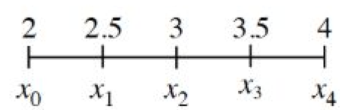
This is read as "The sum of n^2 from $n = 1$ to 3." In this case the sum is equal to 14.

n^2 is the **argument** of the summation. n is the **index**. n must be integer values only.

Using Subscript Notation

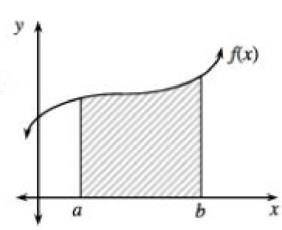
When an interval is broken up into parts, **subscript notation** can be used to represent various values in the interval. For example, if the interval $2 \le x \le 4$ is divided into four equal pieces, the division endpoints will be called x_0, x_1, x_2, x_3, and x_4. Subscript notation is a way to index non-integer and non-consecutive numbers with consecutive integer values. This makes referencing non-integer and non-consecutive numbers more efficient.

In this example, $x_0 = 2$, $x_1 = 2.5$, $x_2 = 3$, $x_3 = 3.5$, and $x_4 = 4$.



Area Under a Curve

Many of the ideas in this textbook and in future mathematics courses are related to the area under a curve. Mathematically, **area under a curve** is the area of the region that is bounded by some function, f, the x-axis, and two vertical line segments located at $x = a$ and $x = b$.

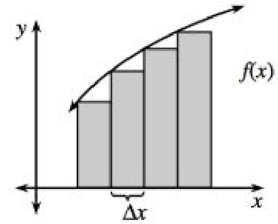


The notation $A(f, a \le x \le b)$ will be used to denote the area under the curve $y = f(x)$ over the interval $a \le x \le b$.

Note: If $f(x) = 9 - 2^x$, then $A(f, 1 \le x \le 3)$ is equivalent to $A(9 - 2x, 1 \le x \le 3)$.

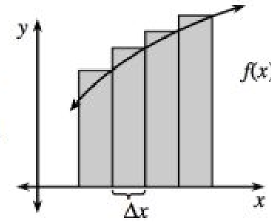
Approximating Area with Rectangles

Left Endpoint Rectangles: The area under the curve $y = f(x)$, approximated by dividing the given interval into n sub-intervals and using the left endpoints of each sub-interval to determine the height of each rectangle is:



$$\sum_{k=0}^{n-1}(f(x_k)\Delta x)$$

Right Endpoint Rectangles: The area under the curve $y = f(x)$, approximated by dividing the given interval into n sub-intervals and using the right endpoints of each sub-interval to determine the height of each rectangle is:



$$\sum_{k=1}^{n}(f(x_k)\Delta x)$$