Math 3

G____

Transformations Toolkit

Name: ______________

Period: A1 A2 A3 B1 B3

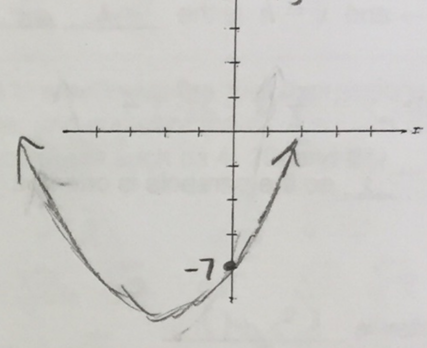
Quadratic Functions

Standard Form: $y = ax^2 + bx + c$, for which $a \neq 0$

This form of the equation shows 3 important characteristics of the graph:

1. The sign (+/-) of a determines the orientation (up or down / positive or negat
2. The value of a is the vertical stretch/compression factor
if $-1 < a < 1$ it compresses if $a > 1$ or $a < -1$ it stretches
3. The value of c is the y-intercept

Example: $y = \frac{1}{2}x^2 + 4x - 7$



For any y-intercept, the x-value is zero

* this form of the equation does not tell us where the vertex is located

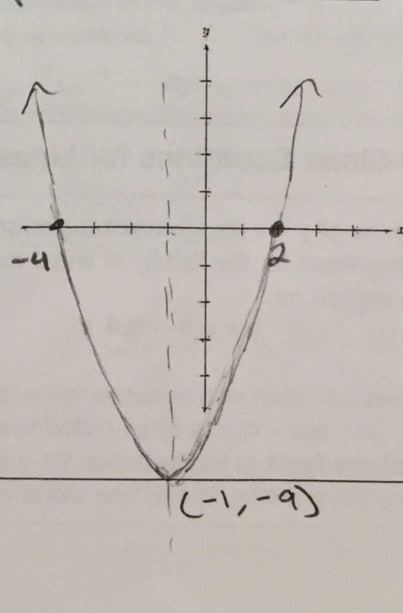
Factored Form: $y = a(x - m)(x - n)$, for which $a \neq 0$

This form of the equation shows 3 important characteristics of the graph:

1. The sign (+/-) of a determines the orientation (up or down / positive or negativ
2. The value of a is the vertical stretch/compression factor.
3. The value of m and n are the x-intercepts

Example: $y = (x - 2)(x + 4)$

For any x-intercept, the y-value equals zero.



$y = (-1-2)(-1+4)$

$y = -3(3) = -9$

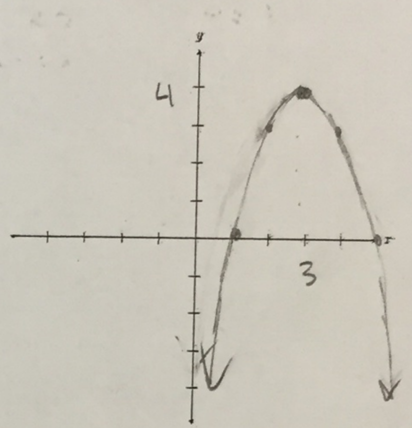
Graphing Form: $y = a(x-h)^2 + k$, for which $a \neq 0$

This form of the equation shows 4 important characteristics of the graph:

1. The sign (+/-) of a determines the orientation
2. The value of a is the vertical stretch / compression
3. The value of h determines the horizontal shift
4. The value of k determines the vertical shift

→ This means (h, k) is the vertex

→ and $x = h$ is the line of symmetry

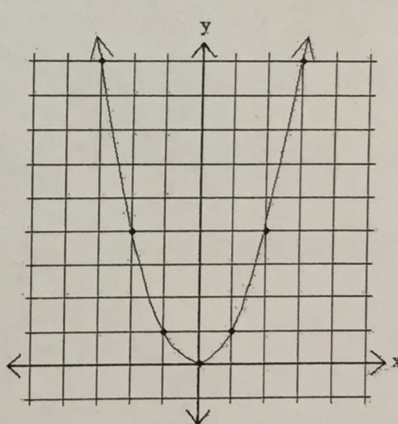


Example: $y = -(x-3)^2 + 4$

a is (−), so the parabola is oriented ↓ and

The vertex is (3,4)

The Parent Function: $y = x^2$



Vertex: (0,0)

Line of Symmetry: X = 0

Orientation: up

Domain: ℝ

Range: y ≥ 0

Shape: parabola "U" shape, curve

Point-Slope Equations for Lines

If you think of $y = x$ as a **parent function**, then the **graphing form** for the family of linear functions can be written as:

$$y = a(x-h) + k$$

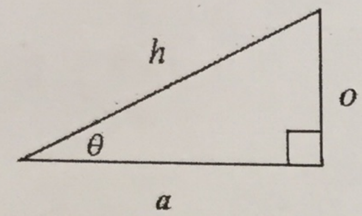
When the equation of a linear function is in the form $y - k = a(x - h)$ it is often called the **point-slope form** of the equation for a line that contains the point (h, k) and has slope a.

For example, if you know a line contains the point (7, –8) and has slope –4, then the equation of the line can be written as:

$y + 8 = -4(x-7)$

Trigonometric Ratios

There are three **trigonometric ratios** you can use to solve for the missing side lengths and angle measurements in any **right triangle.** In the triangle below, when the sides are described relative to the angle θ, the opposite leg is *y* and the adjacent leg is *x*. The hypotenuse is *h* regardless of which acute angle is used.



$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{o}{a}$

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{o}{h}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a}{h}$

Soh Cah Toa

$\cos = \frac{o}{h}$ $\cos = \frac{a}{h}$ $\text{Tan} = \frac{o}{a}$

Rewriting Radical Expressions

Before calculators were readily available, people found it convenient to rewrite square root expressions in a simplified form to make it easier to perform calculations, combine, and compare them. A square root is simplified when there are no more perfect square factors (square numbers such as 4, 25, and 81) under the radical sign.

Example:

$\sqrt{72}$ can be rewritten as $\sqrt{36}\cdot\sqrt{2}$, or as $\sqrt{9}\cdot\sqrt{8}$ and you still get the same result when you simplify completely.

$\sqrt{72}$
$= \sqrt{36}\cdot\sqrt{2}$
$= 6\sqrt{2}$

or

$\sqrt{72}$
$= \sqrt{9}\cdot\sqrt{8}$
$= 3\cdot\sqrt{4}\cdot\sqrt{2}$
$= 3\cdot 2\sqrt{2}$
$= 6\sqrt{2}$

Verify with your calculator that both $6\sqrt{2}$ and $\sqrt{72} \approx 8.485$. ✓

Example: $\sqrt{72} + \sqrt{18}$

To add terms with radicals, the radicals need to be alike. Verify your simplification with your calculator.

$\sqrt{72} + \sqrt{18}$
$6\sqrt{2} + \sqrt{9}\cdot\sqrt{2}$
$6\sqrt{2} + 3\sqrt{2} = 9\sqrt{2}$

It is difficult to estimate the value of a number with a radical in the denominator, and it is also difficult to combine it with or compare it to other numbers. For these reasons, it is sometimes helpful to **rationalize the denominator** so that no radical remains in the denominator.

Example: $\frac{10}{\sqrt{2}}$

First, multiply the numerator and denominator by the radical in the denominator. Since $\frac{\sqrt{2}}{\sqrt{2}} = 1$, this does not change the value of the expression.

$\frac{10}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

After multiplying, notice that the denominator no longer has a radical, since $\sqrt{2}\cdot\sqrt{2} = 2$.

Often, the product can be further simplified.

Imaginary and Complex Numbers
The imaginary number *i* is defined as the square root of –1, so $i = \sqrt{-1}$. Therefore $i^2 = -1$, and the two solutions of the equation $x^2 + 1 = 0$ are x = i and -i.
Real Numbers: The collection of all positive & negative numbers (integers, rational #s, irrational #s) π, 2, -2, 1/4, .25 etc
Imaginary Numbers: Multiplying i by any possible real # except 0 Imaginary #s are not positive, negative or zero.
Complex Numbers: can be written in the form a+bi where a & b are real #s. (ex: 2+i). Real #s are considered complex is b=0 & imaginary #s are complex if a=0

Equations in Graphing Form for Families

If y = *f*(*x*) is an equation for a parent graph, then the equation in graphing form for the family of functions with similar characteristics as *f*(*x*) can be written as:

$$y = a \cdot f(x - h) + k$$

We call (*h*, *k*) the **locator point** because it helps us to locate the transformed graph. The parameters transform the parent graph as follows:

- **horizontally translated** by the value of h
- **vertically translated** by the value of h
- **vertically stretched** if the absolute value of $|a| > 1$
- **vertically compressed** if the absolute value of $|a| < 1$
- **reflected** across the *x*-axis if a is negative

You should be familiar with the following families of functions:

Parent	Family	Graphing Form
$y = x$	Linear	$y = a(x-h) + k$
$y = \lvert x \rvert$	Absolute Value	$y = a\lvert x-h \rvert + k$
$y = x^2$	Quadratic (Parabola)	$y = a(x-h)^2 + k$
$y = x^3$	Cubic	$y = a(x-h)^3 + k$
$y = \frac{1}{x}$	Rational (Hyperbola)	$y = a\left(\frac{1}{x-h}\right) + k$
$y = \sqrt{x}$	Square Root	$y = a\sqrt{x-h} + k$