Precalculus Honors

Trigonometric Functions And Equations Toolkit

Name: ____________________

Date: ____________________

Period:____

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Graphs of $y = \sin(x)$ and $y = \cos(x)$

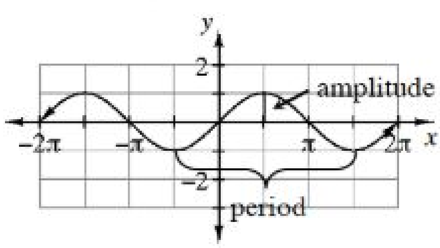
The functions $y = \sin(x)$ and $y = \cos(x)$ are periodic. Intuitively, this means that they "repeat" themselves. You can take a specific piece of the function (a **cycle**) and make the entire graph by copying the piece endlessly to the right and left. The graphs of periodic functions have translation symmetry because you can translate the curve to the left or right and have it land on top of itself.

The horizontal length of one complete cycle of the graph is called the **period**. This is the smallest distance the graph can be shifted horizontally in order to coincide with itself.

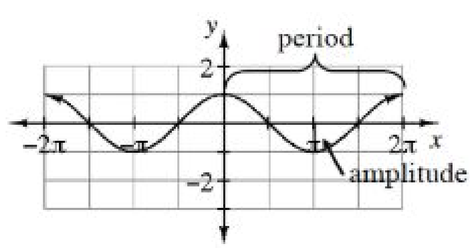
The **amplitude** of the graph of a periodic function is one-half the distance between the highest and lowest points.

For both the sine and cosine functions, the *domain* is all real numbers and the *range* is $-1 \le y \le 1$.

$y = \sin(x)$



$y = \cos(x)$



The 5-Point Method

When graphing a sine or cosine function, it helps to locate five key points that describe the graph. These points are shown on the graph as black dots. But first you need to identify the range of the graph.

A. Start by using the vertical shift to sketch the midline.

B. Use the amplitude to determine how far above and below the midline the graph will stretch.

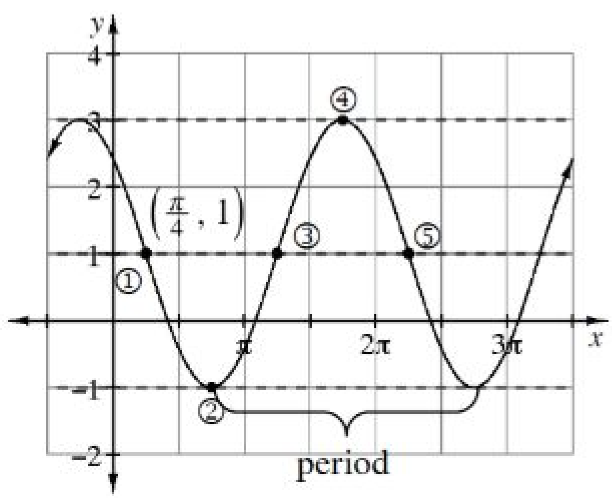
C. Determine where the graph begins (including the horizontal shift) and the direction it is headed. Then locate the five key points on the graph as explained in the following example.

Example: $y = -2\sin\left(x - \frac{\pi}{4}\right) + 1$

A. The vertical shift is + 1, so the midline is $y = 1$.

B. The amplitude = 2, so the range is $-1 \leq y \leq 3$.

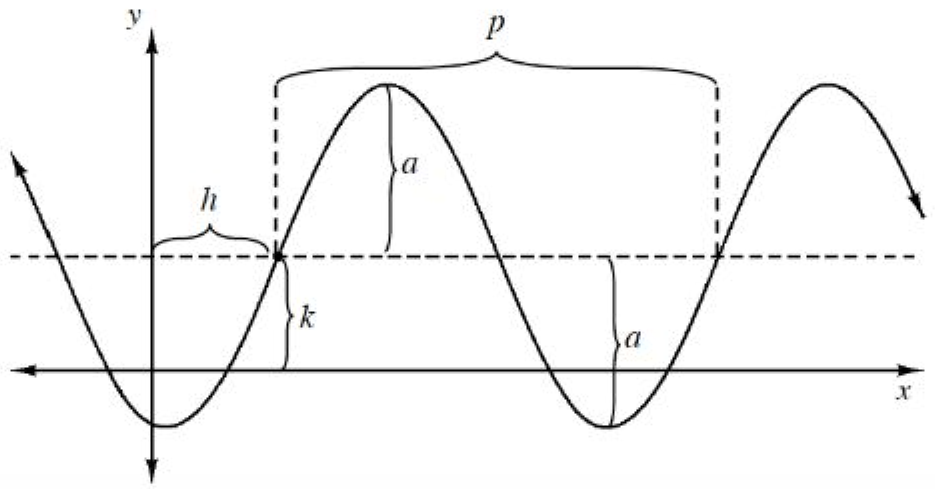
C. The graph is shifted to the right $\frac{\pi}{4}$ units and is a sine function so it begins in the middle. ①
The graph is reflected over the x-axis so it decreases to start. ②
The graph then goes back to the middle. ③
The graph then goes to the top. ④
The graph goes back to the middle to complete one cycle. ⑤



To sketch more of the graph, repeat the steps starting at the end of the first cycle.

General Equation for a Sine Function

A general equation for a **sine function** is $y = a \sin[b(x - h)] + k$.



In the general equation:

A **cycle** of a periodic function is a connected piece of the graph of the function that is one complete repetition of the pattern.

The **amplitude** (half of the vertical distance between the highest and the lowest points) is ***a***.

The **period** is the horizontal length of one complete cycle. It is labeled ***p*** on the graph. To determine the period, use either $p = \frac{2\pi}{b}$ or $pb = 2\pi$.

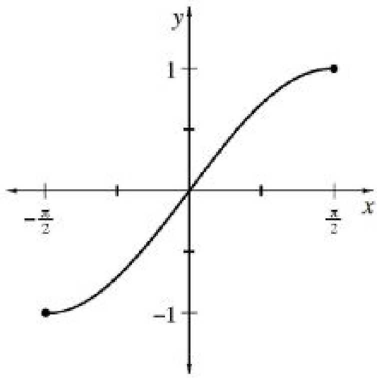
The **horizontal shift** is ***h***.

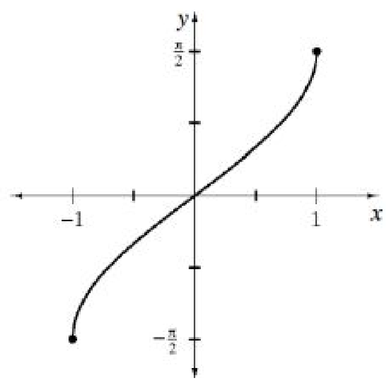
The **vertical shift** is ***k***. The **midline** is $y = k$.

Inverse Trigonometric Functions

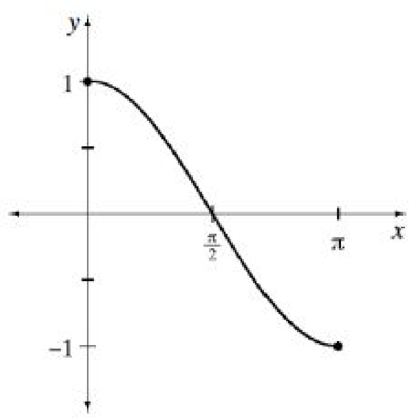
For an inverse of a trigonometric function to be a function, the domain of the original function must be restricted. For each function, the restriction is different.

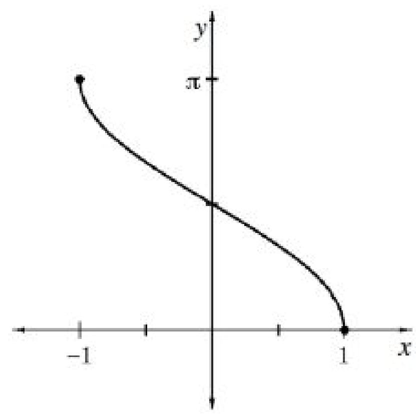
For $y = \sin(x)$, restricting the domain to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ leads to the inverse function $y = \sin^{-1}(x)$, with domain $-1 \leq x \leq 1$ and range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.





For $y = \cos(x)$, restricting the domain to $0 \leq x \leq \pi$ leads to the inverse function $y = \cos^{-1}(x)$, with domain $-1 \leq x \leq 1$ and range $0 \leq y \leq \pi$.





For $y = \tan(x)$, the domain restriction is $-\frac{\pi}{2} < x < \frac{\pi}{2}$, so the domain for inverse tangent, $y = \tan^{-1}(x)$, is all real numbers and the range is $-\frac{\pi}{2} < y < \frac{\pi}{2}$.